Angular Momentum: A Complete Overview

1 Introduction

Angular momentum is a crucial concept in physics that describes the rotational motion of an object around a point or axis. It is a vector quantity, meaning it has both magnitude and direction, and is conserved in isolated systems. Angular momentum plays a vital role in understanding systems ranging from spinning tops to planetary orbits, as well as particles moving in straight lines.

2 Definition of Angular Momentum

For a single particle, the angular momentum $\vec{L}$ with respect to a point is defined as:

$$\vec{L} = \vec{r} \times \vec{p}$$

where:

- $\vec{r}$ is the position vector of the particle relative to the point of reference.
- $\vec{p} = m\vec{v}$ is the linear momentum of the particle (mass m times velocity $\vec{v}$).
- $\times$ denotes the cross product, which means the magnitude of angular momentum depends on the perpendicular distance between $\vec{r}$ and $\vec{p}$.

The magnitude of the angular momentum is given by:

$$L = rp\sin(\theta)$$

where:

- r is the magnitude of the position vector $\vec{r}$,
- p is the magnitude of the linear momentum $\vec{p}$,
- θ is the angle between the position vector $\vec{r}$ and the momentum vector $\vec{p}$.

3 Frame Dependence of Angular Momentum

Angular momentum is a **frame-dependent** quantity, meaning it varies depending on the reference point or coordinate system from which it is measured. The position vector $\vec{r}$ is drawn from the origin of the chosen frame to the particle, so changing the observer's position or the frame of reference alters the value of angular momentum.

For instance, if an observer is moving relative to the particle or located at a different origin, the position vector $\vec{r}$ and the velocity $\vec{v}$ of the particle may appear differently. Therefore, different observers may calculate different angular momenta for the same system.

However, **the conservation of angular momentum** holds true within any given inertial reference frame. So while the value of angular momentum can change with the frame, the laws governing angular momentum remain consistent.

4 Angular Momentum of a Particle Moving in a Straight Line

You might wonder how a particle moving in a straight line can have angular momentum. Let's break it down:

When an object moves in a straight line, its *linear motion* still results in angular momentum if measured relative to a point that is not on the path of the object. The angular momentum is given by the cross product:

$$\vec{L} = \vec{r} \times \vec{p}$$

Even if the motion is linear, as long as the position vector $\vec{r}$ and momentum vector $\vec{p}$ are not parallel (i.e., the particle doesn't pass through the reference point), the angular momentum will be non-zero.

4.1 Example

Imagine a particle moving along the x-axis at a constant speed. If we take the origin of the reference frame below the path of the particle (say at the origin of a 2D Cartesian plane), the position vector $\vec{r}$ is directed from the origin to the particle, and the velocity vector $\vec{v}$ is parallel to the x-axis.

- **Position vector ($\vec{r}$)**: The vector from the reference point (origin) to the particle.
- **Velocity vector ($\vec{v}$)**: The vector representing the velocity of the particle along the x-axis.
- **Angular momentum vector ($\vec{L}$)**: The cross product $\vec{r} \times \vec{p}$ results in a vector pointing perpendicular to the plane of motion. If we consider the

motion to be in the xy-plane, the angular momentum vector points into or out of the page.

This scenario shows that even in linear motion, the object has angular momentum relative to a point outside its path.

5 Diagram: Angular Momentum in Linear Motion

Consider the following 2D diagram, which shows a ball moving parallel to the x-axis. The diagram highlights the position vector $\vec{r}$, the velocity vector $\vec{v}$, and the angular momentum vector $\vec{L}$.

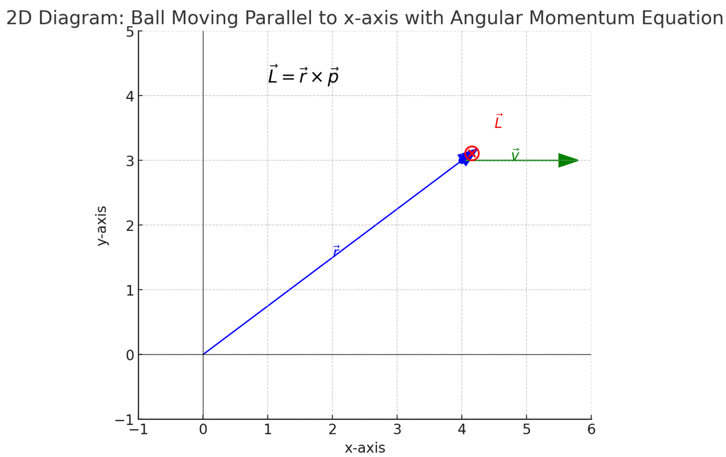

Figure 1: 2D Diagram showing a ball's position, velocity, and angular momentum vector.

- The ball is moving along a horizontal straight line above the x-axis.
- The **position vector** ($\vec{r}$) is drawn from the origin to the ball.
- The **velocity vector** ($\vec{v}$) is pointing along the x-axis, indicating the direction of the ball's motion.
- The **angular momentum vector** ($\vec{L}$) is perpendicular to the plane of motion, represented by a cross ($\otimes$), indicating it's directed into the page.

The equation describing angular momentum is:

$$\vec{L} = \vec{r} \times \vec{p}$$

6 Conclusion

Angular momentum is a vector quantity that depends on both the position and momentum of an object relative to a reference point. Its frame-dependent nature means that its value can change based on the observer's position, but conservation laws still hold in each reference frame. Even objects moving in straight lines possess angular momentum, as long as the reference point is not on their path. Understanding angular momentum in both rotational and linear motion is essential in many areas of physics, from classical mechanics to quantum theory.