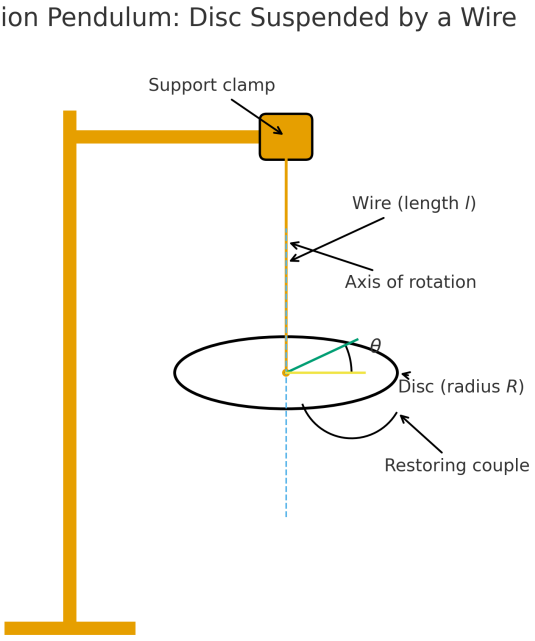

Figure 1: Caption

Experiment: Determination of Moment of Inertia of a Disc using a Torsion Pendulum

Aim:
To determine the **moment of inertia of a disc** about its central axis using a torsion pendulum, and to estimate the experimental error using standard deviation.

Apparatus Required

Torsion pendulum setup, circular disc, suspension wire, stopwatch, meter scale, screw gauge, vernier calipers.

Theory

When a disc suspended by a thin wire is twisted through a small angle and released, it executes torsional oscillations. Let

- θ = angular displacement,
- C = torsional couple per unit twist,
- I = moment of inertia of the disc about the axis of suspension.

Then the restoring couple is proportional to the twist:

$$\text{Restoring couple} = -C\theta.$$

The equation of motion is

$$I\frac{d^2\theta}{dt^2} = -C\theta,$$

or

$$\frac{d^2\theta}{dt^2} + \frac{C}{I}\theta = 0.$$

This represents simple harmonic motion with angular frequency

$$\omega = \sqrt{\frac{C}{I}}.$$

Hence, the time period is

$$T = 2\pi\sqrt{\frac{I}{C}} \quad \Rightarrow \quad \boxed{I = \frac{CT^2}{4\pi^2}}.$$

Torsional Constant of the Wire

For a wire of length l, radius r, and rigidity modulus η,

$$C = \frac{\pi\eta r^4}{2l}.$$

Therefore,

$$\boxed{I = \frac{\eta r^4 T^2}{8\pi l}}.$$

Procedure

1. Suspend the disc horizontally using the suspension wire through its centre.
2. Twist the disc gently through a small angle and release it to perform torsional oscillations.
3. Measure the time for 10 oscillations using a stopwatch.
4. Repeat the measurement three times and find the mean time.
5. Measure the length l and diameter d of the wire using a metre scale and screw gauge.
6. Compute the radius $r = d/2$.
7. Calculate the mean time period $T = t_m/10$.
8. Compute the standard deviation of time and its propagation into T and I.

Observations

Trial No.	Time for 10 oscillations (s)	Remarks
1		
2		
3		

$$l = \ldots \text{ m}, \quad d = \ldots \text{ mm}, \quad r = \frac{d}{2}, \quad \eta = \ldots \text{ N·m}^{-2}.$$

Calculations

1. Mean Time

$$t_m = \frac{t_1 + t_2 + t_3}{3}.$$

2. Standard Deviation of Time

$$\sigma_t = \sqrt{\frac{(t_1 - t_m)^2 + (t_2 - t_m)^2 + (t_3 - t_m)^2}{3 - 1}}.$$

3. Mean Time Period

$$T = \frac{t_m}{10}.$$

4. Error in Time Period

$$\sigma_T = \frac{\sigma_t}{10}.$$

5. Moment of Inertia

$$I = \frac{\eta r^4 T^2}{8\pi l}.$$

6. Error in Moment of Inertia (approximate propagation)

$$\frac{\sigma_I}{I} = 2\frac{\sigma_T}{T}, \qquad \text{so that} \qquad \sigma_I = I \times 2\frac{\sigma_T}{T}.$$

Result

$$\boxed{I = (\text{value}) \pm \sigma_I \text{ kg·m}^2.}$$

Hence, the moment of inertia of the given disc about its axis of suspension is determined.

Error Analysis

- Random error estimated using standard deviation of repeated time measurements.
- Percentage error in time period:

$$\frac{\sigma_T}{T} \times 100\%.$$

- Percentage error in I:

$$\frac{\sigma_I}{I} \times 100\%.$$

- Main sources of error:
  1. Friction at the suspension point.
  2. Air damping.
  3. Non-uniform thickness of the wire.
  4. Human reaction time in stopwatch measurement.

Precautions

1. Keep the amplitude of oscillation small.
2. Do not twist the wire beyond its elastic limit.
3. Ensure the disc oscillates freely without wobbling.
4. Record time for at least 10 oscillations.
5. Avoid vibrations and air currents during observation.

Conclusion

The moment of inertia of the disc about its central axis was determined using the torsion pendulum method. The experimental value agrees with theoretical expectations within the limits of error, as estimated using standard deviation.