Experiment: Determination of the Length of a Simple Pendulum Equivalent to a Symmetric Compound Pendulum

Aim:
To determine the length of a simple pendulum that has the same time period as a given symmetric compound pendulum and to estimate the error using standard deviation.

Apparatus Required

Compound pendulum (uniform bar with holes), knife-edge support, stopwatch, meter scale, vernier calipers, spirit level, and stand.

Theory

For a compound pendulum of mass M, moment of inertia I about the axis of suspension, and distance h between the axis of suspension and the center of gravity (C.G.), the time period is given by

$$T = 2\pi\sqrt{\frac{I}{Mgh}}.$$

By the parallel axis theorem,

$$I = I_G + Mh^2,$$

where I_G is the moment of inertia about an axis through the C.G.

Substituting,

$$T = 2\pi\sqrt{\frac{I_G + Mh^2}{Mgh}} = 2\pi\sqrt{\frac{k^2 + h^2}{gh}},$$

where k is the radius of gyration about the C.G.

For a simple pendulum of length L,

$$T = 2\pi\sqrt{\frac{L}{g}}.$$

Equating the two expressions for equal periods,

$$\frac{L}{g} = \frac{k^2 + h^2}{gh} \quad \Rightarrow \quad \boxed{L = h + \frac{k^2}{h}}.$$

Thus, the equivalent length of the simple pendulum corresponding to the given compound pendulum is

$$L = h + \frac{k^2}{h}.$$

Procedure

1. Suspend the compound pendulum from a knife edge near one end and set it oscillating through a small amplitude.
2. Measure the time for 20 complete oscillations using a stopwatch.
3. Repeat the measurement three times and take the mean value.
4. Calculate the time period $T = \frac{t}{20}$.
5. Measure the distance h of the knife edge from the C.G. using a meter scale.
6. Repeat the above steps for different suspension points on both sides of the C.G.
7. Plot a graph between T^2 (on the y-axis) and h (on the x-axis).
8. The graph will be a U-shaped curve. For equal time periods on either side of the C.G., let the distances be h_1 and h_2.
9. Compute $k^2 = h_1 h_2$ and $L = h_1 + h_2$.

Observations

Sl. No.	h (cm)	Time for 20 oscillations t_1 (s)	t_2 (s)	t_3 (s)	Mean t_m (s)	$T = t_m/20$ (s)
1						
2						
3						
4						

Calculations

1. Mean Time

$$t_m = \frac{t_1 + t_2 + t_3}{3}$$

2. Standard Deviation

The standard deviation of the time measurements is given by:

$$\sigma_t = \sqrt{\frac{(t_1 - t_m)^2 + (t_2 - t_m)^2 + (t_3 - t_m)^2}{3 - 1}}$$

3. Error in Time Period

Since $T = \frac{t_m}{20}$, the error in T is

$$\sigma_T = \frac{\sigma_t}{20}.$$

4. Error in T^2

For T^2, the propagated error is

$$\sigma_{T^2} = 2T\,\sigma_T.$$

5. Equivalent Length

From the T^2–h graph, for equal time periods we obtain:

$$h_1 = \ldots \text{ cm}, \quad h_2 = \ldots \text{ cm}.$$

Then,

$$k^2 = h_1 h_2, \qquad L = h_1 + h_2.$$

Error Analysis

- The random error in time measurement is represented by the standard deviation $\sigma_T = \sqrt{\overline{T_i - \overline{T_i}}}$ and $\sigma_{T^2} = \sqrt{\overline{(T^2)_i - \overline{(T^2)_i}}}$.
- The percentage error in time period is given by
$$\frac{\sigma_T}{T} \times 100\%.$$
- The percentage error in T^2 is
$$\frac{\sigma_{T^2}}{T^2} \times 100\%.$$
- Systematic errors may arise due to air resistance, friction at knife edges, or misalignment of the axis.

Result

$$\boxed{L = (\text{value})\ \text{cm}}$$

Hence, the length of the simple pendulum equivalent to the given symmetric compound pendulum is determined. The experimental error is quantified using standard deviation.

Precautions

1. The oscillations should be of small amplitude (less than 5°).
2. The knife edges should be sharp and rest firmly on the support.
3. The pendulum should oscillate in a single vertical plane.
4. Take time for at least 20 oscillations to reduce reaction time error.
5. Avoid air currents.

Sources of Error

1. Difficulty in locating the center of gravity accurately.
2. Friction at the knife edges.
3. Parallax error in measuring distances.
4. Reaction time error in stopwatch readings.

Conclusion

The equivalent length of the simple pendulum was determined from the T^2–h graph. The experimentally obtained value of L agrees well with the theoretical expression

$$L = h_1 + h_2,$$

and the uncertainty was estimated using standard deviation analysis.

Discussion on h, h_1, and h_2

Let the distance between the axis of suspension and the centre of gravity (C.G.) of the compound pendulum be denoted by h. For different holes or knife edges along the length of the pendulum, the value of h changes, and consequently the time period T also changes.

Variation of T^2 with h

The theoretical relation is

$$T^2 = \frac{4\pi^2}{g}\left(h + \frac{k^2}{h}\right),$$

where k is the radius of gyration about the C.G. and g is the acceleration due to gravity.

The function $T^2(h)$ increases for both very small and very large values of h, and has a **minimum** at $h = k$. Thus, a plot of T^2 versus h gives a U-shaped curve (not a parabola).

Meaning of h_1 and h_2

If a horizontal line is drawn to represent a constant time period T, it intersects the T^2–h curve at two points, say h_1 and h_2, such that

$$T^2(h_1) = T^2(h_2).$$

This means that the pendulum will have the **same time period** when suspended at distances h_1 and h_2 from the C.G. on either side.

Mathematically, this equality leads to the relation

$$h_1 h_2 = k^2.$$

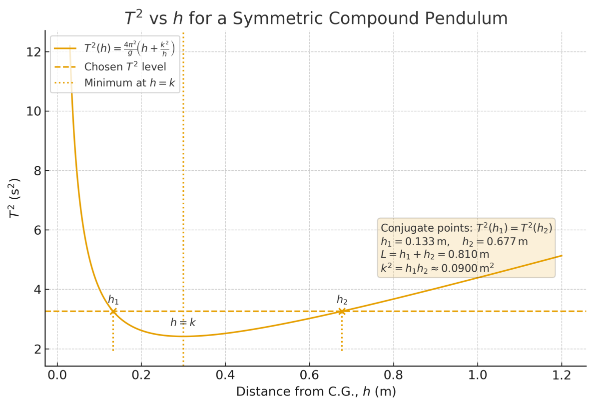

Figure 1: The graph to be drawn

Physical Interpretation

- The quantity h represents the distance from the C.G. to the point of suspension.
- The two values h_1 and h_2 correspond to two distinct suspension points — one on each side of the C.G. — that give the same time period.
- These two points are known as **conjugate points**.
- The length of a simple pendulum having the same time period as the compound pendulum is given by

$$\boxed{L = h_1 + h_2.}$$

Hence, the significance of h_2 is that it represents the second suspension point (on the opposite side of the C.G.) which, together with h_1, defines the equivalent simple pendulum length L. This equivalence shows that the compound pendulum behaves as if its mass were concentrated at the midpoint of the equivalent simple pendulum of length $L = h_1 + h_2$.