Lecture Notes: Quantum Theory of Simple Gases and Ideal Gas in Quantum Microcanonical Ensemble

Based on Pathria
Statistical Mechanics, 3rd Edition

1. Quantum Theory of Simple Gases (Chapter 7)

1.1 Motivation

Classical statistical mechanics breaks down at low temperatures or high densities. In such regimes, quantum effects dominate, and particles obey:

- **Bose-Einstein statistics** (bosons)
- **Fermi-Dirac statistics** (fermions)

1.2 Distinguishability and Quantum Statistics

Quantum particles are *indistinguishable*. This leads to the following occupation rules:

- **Fermions**: $n_i = 0$ or 1 (Pauli exclusion principle)
- **Bosons**: $n_i = 0, 1, 2, \ldots$

1.3 Occupation Number Representation

Let ε_i denote single-particle energy levels. Then:

$$N = \sum_i n_i$$
$$E = \sum_i n_i \varepsilon_i$$

The many-particle quantum state is specified by the set $\{n_i\}$.

1.4 Counting Microstates

The number of microstates depends on the symmetrization or antisymmetrization of the many-particle wavefunction. The total number of distinct configurations is constrained by N and E.

2. Ideal Gas in Quantum Microcanonical Ensemble (Section 7.2)

2.1 Ensemble Description

The system is isolated with fixed:

- Energy E
- Volume V
- Particle number N

This is the quantum analog of the classical microcanonical ensemble.

2.2 Quantum Microstates

Each microstate corresponds to an occupation number configuration $\{n_i\}$ satisfying:

$$\sum_i n_i = N$$
$$\sum_i n_i \varepsilon_i = E$$

2.3 Entropy

The entropy is defined by:

$$S(E, V, N) = k_B \ln \Omega(E, V, N)$$

where Ω is the number of microstates satisfying the constraints.

2.4 Thermodynamic Limit

In the limit $N \to \infty$, energy levels become densely packed, and sums over states are replaced by integrals. This allows thermodynamic quantities to be computed from smooth functions.

2.5 Summary of Differences

Feature	**Classical**	**Quantum**
Particles	Distinguishable	Indistinguishable
States	Phase space points	Occupation number configs
Symmetry	None	Symmetrization/Antisymmetrization
Allowed n_i	Any	$0, 1$ (fermions); $0, 1, 2, ...$ (bosons)

For large systems, the quantum microcanonical ensemble gives rise to classical thermodynamic behavior, but only after accounting for quantum statistics.