Detailed Notes on Magnetic Behavior of Ideal Fermi Gas and Electron Gas in Metals

1 Pauli Paramagnetism

Consider a gas of non-interacting fermions with intrinsic magnetic moment μ in an external magnetic field $\mathbf{B}$. Assume the spin of the particle is $\frac{1}{2}$, so the magnetic moment μ can be either parallel or antiparallel to $\mathbf{B}$.

The energy of a particle is:

$$E = \frac{p^2}{2m} - \mu B \quad \text{or} \quad E = \frac{p^2}{2m} + \mu B \tag{1}$$

for parallel or antiparallel alignment, respectively.

At $T = 0$, fermions fill energy levels up to the Fermi energy ϵ_F. The number of particles in each spin state is:

$$N_+ = \frac{4\pi V}{3h^3}(2m)^{3/2}(\epsilon_F - \mu B)^{3/2} \tag{2}$$

$$N_- = \frac{4\pi V}{3h^3}(2m)^{3/2}(\epsilon_F + \mu B)^{3/2} \tag{3}$$

Net magnetization:

$$M = \mu(N_+ - N_-) \approx \frac{4\pi V}{3h^3}(2m)^{3/2} \cdot 3\mu^2 B \cdot \epsilon_F^{1/2} \tag{4}$$

Hence, the susceptibility is:

$$\chi = \left.\frac{M}{VB}\right|_{B\to 0} = \frac{2n\mu^2}{\epsilon_F} \tag{5}$$

This is the **Pauli paramagnetic susceptibility**, which is independent of T for low temperatures.

2 Landau Diamagnetism

For a charged particle in a magnetic field $\mathbf{B} = B\hat{z}$, the quantized cyclotron motion in the xy-plane gives Landau levels:

$$E_{j,p_z} = \hbar\omega_c\left(j + \frac{1}{2}\right) + \frac{p_z^2}{2m} \tag{6}$$

where $\omega_c = \frac{eB}{mc}$ is the cyclotron frequency.

The degeneracy per Landau level is:

$$g_j = \frac{eB}{hc} L_x L_y \tag{7}$$

The grand partition function becomes:

$$\ln \mathcal{Z} = \frac{VeB}{h^2 c} \int \frac{dp_z}{2\pi\hbar} \sum_j \ln\left(1 + ze^{-\beta E_{j,p_z}}\right) \tag{8}$$

For weak fields and high T, using Euler-Maclaurin expansion:

$$\chi = -\frac{e^2\hbar^2}{3m^2c^2} \cdot \frac{n}{kT} \tag{9}$$

This is **Landau diamagnetism**, negative in sign and also obeys Curie law at high T. At $T \to 0$, susceptibility saturates.

3 Total Susceptibility

Combining Pauli and Landau effects:

$$\chi_{\text{total}} = \chi_{\text{Pauli}} + \chi_{\text{Landau}} = \frac{2n\mu_B^2}{\epsilon_F} - \frac{ne^2\hbar^2}{3m^2c^2\epsilon_F} \tag{10}$$

At low T, both terms are independent of temperature.

4 Electron Gas in Metals

Drude-Lorentz model failed to explain:

- Small specific heat contribution from electrons
- Weak and T-independent magnetic susceptibility

Sommerfeld theory applies Fermi-Dirac statistics:

$$C_V = \frac{\pi^2}{2} Nk \frac{T}{T_F} \tag{11}$$

$$\chi_P = \frac{3n\mu_B^2}{2\epsilon_F} \quad \text{(Pauli)} \tag{12}$$

Low temperature specific heat:

$$C_V = \gamma T + \beta T^3 \tag{13}$$

- γ from electrons, linear in T
- β from lattice, Debye T^3 law

Magnetism: Weak field susceptibility = Pauli (paramagnetic) + Landau (diamagnetic). Observed susceptibility is consistent with this.

Transport properties: Wiedemann–Franz law:

$$\frac{K}{\sigma T} = \text{Lorenz number} = \frac{\pi^2}{3}\left(\frac{k}{e}\right)^2 \tag{14}$$

5 Thermionic and Photoelectric Emission

Electrons in a metal are trapped in a potential well. Escape condition:

$$\frac{1}{2}mu_z^2 > W \tag{15}$$

Rate of emission similar to effusion:

$$R = \int_{\sqrt{2W/m}}^{\infty} du_z \cdots \tag{16}$$

Modified by a reflection coefficient r: actual emission $\sim (1-r)R$.