Lecture Notes: Counting Microstates for Classical, Fermi, and Bose Particles

Based on Pathria *Statistical Mechanics*, 3rd Edition

1. Introduction

A central problem in statistical mechanics is to count the number of microstates corresponding to a given macrostate. The counting rules differ for:

- Classical distinguishable particles
- Bosons (indistinguishable, symmetric states)
- Fermions (indistinguishable, antisymmetric states)

We consider a system where N particles are distributed among G energy levels.

2. Classical (Maxwell-Boltzmann) Statistics

Assumption: Particles are distinguishable, and there is no restriction on occupation number.

Number of configurations:

$$W_{MB} = \frac{G^N}{N!}$$

The factor $1/N!$ corrects for overcounting due to indistinguishability in classical limit.

3. Bose-Einstein Statistics (Bosons)

Assumption: Particles are indistinguishable, and multiple occupancy of a single state is allowed.

Number of configurations:

$$W_{BE} = \frac{(N+G-1)!}{N!(G-1)!}$$

This is the number of integer solutions to:

$$\sum_{i=1}^{G} n_i = N, \quad n_i \in \{0, 1, 2, \ldots\}$$

4. Fermi-Dirac Statistics (Fermions)

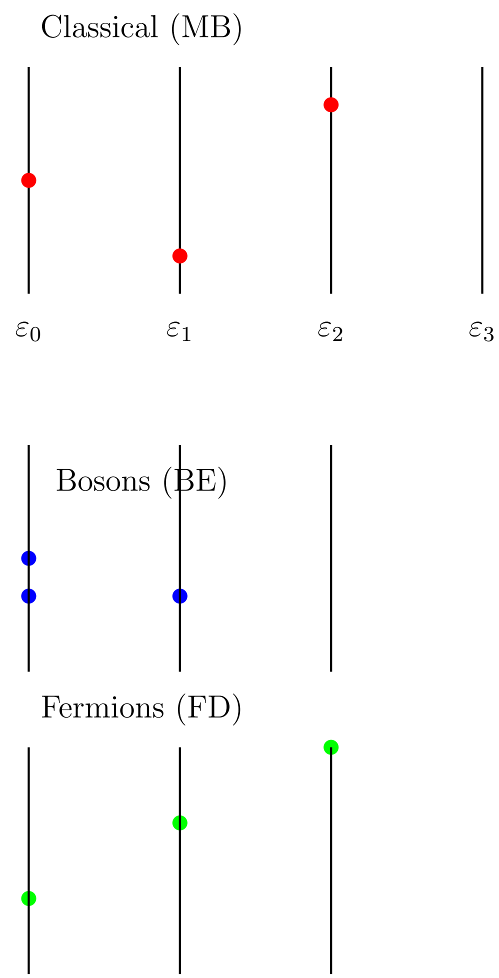
Assumption: Particles are indistinguishable and obey the Pauli exclusion principle ($n_i = 0$ or 1).

Number of configurations:

$$W_{FD} = \binom{G}{N}$$

This is the number of ways to choose N distinct single-particle states out of G.

5. Visual Illustration



6. Summary

Statistics	Indistinguishable?	Occupancy	Formula for W
Classical (MB)	No	Any	$\frac{G^N}{N!}$
Bose-Einstein	Yes	$n_i = 0, 1, 2, ...$	$\frac{(N+G-1)!}{N!(G-1)!}$
Fermi-Dirac	Yes	$n_i = 0$ or 1	$\binom{G}{N}$

7. Logic Behind the Counting

- **Classical (MB):** Each particle can go into any of the G states. Since they are distinguishable, this yields G^N configurations. However, since particles are physically identical in nature, we divide by $N!$ to avoid overcounting.
- **Bose-Einstein (BE):** Particles are indistinguishable, and more than one particle can occupy a single state. The problem reduces to placing N indistinguishable balls into G distinguishable boxes, which is equivalent to the number of non-negative integer solutions of $n_1 + n_2 + \cdots + n_G = N$. contact me if you want to know more
- **Fermi-Dirac (FD):** Particles are indistinguishable and obey the Pauli exclusion principle. Each state can be occupied by at most one particle. So, we simply choose N out of G available states.

1. Distinguishability vs. Physical Identity

In statistical mechanics, we often say that classical particles are **distinguishable** even though they may be **physically identical**. This subtlety arises from the nature of classical versus quantum descriptions:

- **Physically identical** means that the particles have the same mass, charge, spin, etc.
- **Distinguishable** in the classical context means that we can, in principle, *label* each particle individually based on its trajectory or initial condition.

Example: In classical mechanics, two identical gas molecules (say, argon atoms) can be tagged as particle 1 and particle 2 by their paths, even if they are of the same species. So exchanging them results in a new microstate. Hence, the total number of configurations is G^N.

However, this leads to an overcounting of microstates when calculating entropy. To correct for this, we divide by $N!$ in the classical Maxwell-Boltzmann statistics:

$$W_{MB} = \frac{G^N}{N!}$$

In contrast, in quantum mechanics, identical particles are *fundamentally indistinguishable* — swapping them does not yield a new physical state.

2. Importance of Degenerate Energy Levels

Energy levels in quantum systems may be **degenerate**, meaning that more than one independent quantum state shares the same energy.

- Let ε be an energy level with degeneracy g.
- Then there are g different states all having energy ε.

- This increases the number of ways particles can be arranged among energy levels.

Why is this important?

- The **density of states** affects the statistical weight of configurations.
- Degeneracy plays a major role in the entropy and partition function.
- For fermions, degeneracy allows multiple states to be filled within the same energy.

Example: In a 3D particle-in-a-box system, the energy depends on quantum numbers:

$$\varepsilon_{n_x,n_y,n_z} \propto n_x^2 + n_y^2 + n_z^2$$

Multiple combinations of (n_x, n_y, n_z) can yield the same total energy, leading to degeneracy.

Summary

- Classical distinguishability is a bookkeeping convention, corrected by $1/N!$.
- Quantum indistinguishability is fundamental, shaping statistical laws.
- Degeneracy of energy levels enhances the number of microstates and must be factored into statistical calculations.